This question paper contains 8+1 printed pages]

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M.A./M.Sc. (Final) Examination, 2022

MATHEMATICS

Paper I (Optional)

Operations Research

Time Allowed : Three Hours

Maximum Marks : 100

Note : Non-Programmable Scientific Calculator is allowed in this paper.

This question paper contains ***three*** *sections as under :*

Section-A

Max. Marks-10

This section contains ***one*** *compulsory question with* ***10*** *parts, having* ***2*** *parts from each unit, short answer in* ***20*** *words for each part. All questions carry equal marks.*

Section-B ***Max. Marks-50***

This section contains ***10*** *questions, having* ***2*** *questions from each unit. Answer* ***five*** *questions (****250*** *words each) selecting* ***one*** *question from each unit. All questions carry equal marks.*

Section-C ***Max. Marks-40***

This section contains ***4*** *descriptive type questions (questions may have sub-divisions) covering all units but not more than* ***one*** *question from each unit. Answer any* ***two*** *questions (****500*** *words each). All questions carry equal marks.*

Section A

1. (*i*) Define the Degeneracy in Simplex Method.

(*ii*) Write the matrix form of unsymmetric Primal-dual problem.

(*iii*) Define the pure and mixed strategies.

(*iv*) What is integer programming ? Explain briefly.

(*v*) Define the 'principle of optimality' in dynamic programming.

(*vi*) Write the difference between PERT and CPM.

(*vii*) Describe the basic characteristics of the Inventory system.

(*viii*) Write the KT conditions for a Minimization NLPP.

(*ix*) Explain Wolfe's method in quadratic programming briefly.

(*x*) What is Queue ? Give an example.

Section B

UNIT-I

2. Solve the following LPP, using simplex method :

Maximize : $Z = 6x_1 + 10x_2 + 8x_3$

Subject to : $2x_1 + 3x_2 \leq 80$

$2x_2 + 5x_3 \leq 100$

$3x_1 + 2x_2 + 4x_3 \leq 150$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$

3. Use duality to solve the following LPP :

Min. : $Z = 10x_1 + 6x_2 + 2x_3$

Subject to : $-x_1 + x_2 + x_3 \geq 1$

$3x_1 + x_2 - x_3 \geq 2$

and $x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$

UNIT-II

4. Solve the following mixed integer programming problem using Gomory's technique :

Maximize : $Z = x_1 + x_2$

Subject to : $3x_1 + 2x_2 \leq 5$

$x_2 \leq 2$

and $x_1, x_2 \geq 0$ and x_1 an integer

5. Consider (two-person zero-sum) game matrix which represents payoff to the player A, find the optimal strategy, if any :

B

A \ B	I	II	III
I	−3	−2	6
II	2	0	2
III	5	−2	−4

UNIT-III

6. A project consists of a series of tasks labelled A, B,.....I, with the following relationship :

A < D, E; B, D < F; C < G; B < H;

F, G < I

(1) Construct the network diagram.

(2) Minimum time of completion of the project, when the time of completion of each tasks is as follows :

Task	A	B	C	D	E	F	G	H	I
Time (days)	23	8	20	16	24	18	19	4	10

7. Assuming that the expected time are normally distributed find the probability of meeting the schedule project date as for given network :

Activity (i, j).	Estimated Durations (Days)		
	Optimistic (t_0)	Most likely (t_m)	Pessimistic (t_p)
(1, 2)	2	5	14
(1, 3)	9	12	15
(2, 4)	5	14	17
(3, 4)	2	5	8
(3, 5)	8	17	20
(4, 5)	6	6	12

Scheduled project completion time is 30 days. Also find the time in which the project can be completed with a probability 0.90.

UNIT-IV

8. Calculate the EOQ and the total variable cost for the following :

Annual demand = 25 units; Unit price = Rs. 2.50; Order cost = Rs. 4.00; Storage cost = 1% per year Interest rate = 12% per year; obsolescene rate = 7% per year.

9. Determine the relative maxima and minima (if any) of the function :

$$f(x_1\ x_2\ x_3) = x_1 + 2x_3 + x_3x_2 - x_1^2 - x_2^2 - x_3^2$$

UNIT-V

10. Solve the following quadratic programming problem : Beats Mr Wolfe's

Minimize :

$$f\ (x_1\ x_2) = x_1^2 - x_1x_2 + 2x_2^2 - x_1 - x_2$$

[Contd....

subject to : $2x_1 + x_2 \leq 1$

$$x_1 \geq 0,\ x_2 \geq 0$$

11. Customers arrive at a sales counter manned by a single person according to a Poisson process with a mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 100 seconds. Find the average waiting time of a customer.

Section C

12. Obtain optimal solution of LPP :

Max. $z = 3x_1 + 5x_2 + 4x_3$

Subject to : $2x_1 + 3x_2 \leq 8$

$$2x_2 + 5x_3 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

and $x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$

Find the permissible change in the value of C_3 and C_4 so that the same solution still remain optimal.

13. Solve by Dynamic programming techniques the problem :

$$\text{Min } Z = y_1^2 + y_2^2 + y_3^2$$

Subject to : $y_1\ y_2\ y_3 = 27$

and $y_1\ y_2\ y_3 > 0$

14. Derive the optimal solution from the Kuhn-Tucker condition for the problem.

$$\text{Minimize } Z = 2x_1 + 3x_2 - x_1^2 - 2x_2^2$$

Subject to : $x_1 + 3x_2 \leq 6$

$$5x_1 + 2x_2 \leq 10$$

and $x_1 \geq 0,\ x_2 \geq 0$

15. Write a short note on M|M|C models and their applications.