UB-1652

M. A. / M. Sc. (Previous) Examination, 2023

MATHEMATICS

Paper - II

REAL AND COMPLEX ANALYSIS

Time allowed : **Three Hours**

Maximum Marks : **100**

*This question paper contains **three** sections as under :*

Section-A खण्ड–अ ***Max. Marks-10***

This section contains one compulsory question with 10 parts, having 2 parts from each unit, short answer in 20 words for each part. All questions carry equal marks.

इस खण्ड में एक अनिवार्य प्रश्न है जिसमें प्रत्येक इकाई से 02 लघु प्रश्न लेते हुये कुल 10 लघु प्रश्न होंगे। प्रत्येक लघु प्रश्न का उत्तर २० शब्दों से अधिक में न हो। सभी प्रश्नों के अंक समान है।

Section-B खण्ड–ब ***Max. Marks-50***

*This section contains **10** questions having 2 questions from each unit. Answer 5 questions (250 words each) selecting one question from each unit. All questions carry equal marks.*

इस खण्ड में प्रत्येक इकाई से 02 प्रश्न लेते हुये कुल 10 प्रश्न है। प्रत्येक इकाई से एक प्रश्न का चयन करते हुये कुल 5 प्रश्नों के उत्तर देने है। प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक में न हो। सभी प्रश्नों के अंक समान है।

Section-C खण्ड–स ***Max. Marks-40***

This section contain 4 descriptive type question (questions may have sub division) covering all units but not more than one question from each unit. Answer any two questions. (500 words each). All questions carry equal marks.

इस खण्ड में 4 वर्णनात्मक प्रश्न होंगे। प्रश्न में भाग भी हो सकते हैं जो सभी इकाईयों में से दिये जायेगें, किन्तु एक इकाई में से एक से अधिक प्रश्न नहीं होगा। कोई दो प्रश्नों के उत्तर दिये जाने है। प्रत्येक प्रश्न का उत्तर 500 शब्दों से अधिक न हो। सभी प्रश्नों के अंक समान हैं।

SECTION - A

1 (i) Define uniform convergence of a series of functions.

(ii) Define Riemann-Stieltjes integral.

(iii) Prove that R is a measurable set.

(iv) Define Borel measurable function.

(v) Define simply and multi connected region.

(vi) Write polar form of Cauchy-Riemann equations.

(vii) State Taylor's theorem for complex function.

(viii) What is maximum modulus principle?

(ix) Define residue at a singularity.

(x) Define analytic continuation.

10×1=10

SECTION - B (10×5=50)

UNIT - I

2 If $f(x)=k$ be a constant function defined on [a, b] and $\alpha(x)$ is a monotonically non decreasing function on [a, b], then prove that $f(x)$ is Riemann-Stieltjes integrable.

OR

3 Show that the series $\sum \frac{(-1)^{n-1}}{n}|x|^n$ is uniformly convergent on [-1, 1].

UNIT - II

4 If $\{En\}$ be a countable collection of sets real numbers then

$$m^*\left\{\bigcup_n En\right\} \le \sum_n m^*(En)$$

OR

5 Prove that the characteristic function ϕ_A of a set A is measurable if and only if A is a measurable set.

UNIT - III

6 Prove that the function $u = e^x(x\cos y - y\sin y)$ satisfies laplaces's equation and find the corresponding analytic function $f(z) = u + iv$.

OR

7 Find a bilinear transformation that maps the points $z = \infty, i, o$ into the points $w = o, i, \infty$.

UNIT - IV

8 State and prove Cauchy's integral formula for simply connected domain.

OR

9 Find the value of

$$\int_{|z|=1} \frac{\sin^6 z}{\left[z - \frac{\pi}{6}\right]^3} dz$$

UNIT - V

10 Find the residue of $f(z) = \frac{z^2 - 2z}{(zH)^2 (z^2 + u)}$ at all its poles in the finite plane.

OR

11 Show that the function $f(z) = 1 + z + z^2 + z^3 + \ldots$ can be continued analytically outside the circle of convergence.

SECTION - C (20×2=40)

12 (i) Prove that the family M of all measurable sets is a σ algebra of sets in $P(R)$.

(ii) Prove that if $\{fn\}$ be a sequence of measurable function converges pointwise to a function f on measurable set E $\left[i.e. \lim_{n \to \infty} fn(x) = f(x)\right]$, then the function f is measurable.

13 (i) Behaviour of power series $\sum \frac{z^{un}}{un+1}$ on the circle of convergence.

(ii) Show that the transformation $w(z+i)^2 = 1$ maps the interior of the circle $|z| = 1$ in the z-plane on the exterior of the parabola $\frac{1}{\rho} = 2(1 - \cos\phi)$ where $w = \rho e^{i\phi}$ in the w-plane.

14 (i) State and prove Schwarz's Lemma.

(ii) State and prove poisson integral formula.

15 (i) State and prove open mapping theorem.

(ii) State and prove Casoratti - Weirstrass theorem.