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M.A./M.Sc. (Previous) Examination, 2018

MATHEMATICS

Paper V

(Discrete Mathematics and Numerical Analysis)

Time Allowed : Three Hours

Maximum Marks : 100

This question paper contains ***three*** *sections as under :*

Section-A खण्ड 'अ' ***Max. Marks-10***

This section contains ***one*** *compulsory question* ***with 10 parts, having 2 parts from each unit, short answer in 20 words for each part. All*** *questions carry* ***equal marks.***

इस खण्ड में ***एक*** *अनिवार्य प्रश्न है जिसमें प्रत्येक इकाई* ***से 2*** *लघु प्रश्न लेते हुए कुल* ***10*** *लघु प्रश्न होंगे। प्रत्येक लघु प्रश्न का उत्तर* ***20*** *शब्दों से अधिक में न हो। सभी प्रश्नों के अंक समान हैं।*

SECTION-A

1. (*i*) State the principle of mathematical induction.

(*ii*) Define directed graph.

(*iii*) Define ordered rooted tree.

(*iv*) Define a lattice.

(*v*) Write iterative formula for Regula-Falsi method.

(*vi*) State least square principle.

(*vii*) What is the difference between direct and iterative methods ?

(*viii*) Define eigenvalue and eigenvector of a matrix.

(*ix*) Write third order Runge-Kutta method.

(*x*) Write a short note on finite difference scheme.

SECTION-B

UNIT-I

2. Prove by the principle of mathematical induction that $9^n - 8n - 1$, is divisible by 64, for all integers $n \geq 2$.

3. Prove that a conneted graph G is an Euler graph if and only if it has no vertex of odd degree.

UNIT-II

4. If h is the height of a balanced complete binary tree on n vertices, then prove that :

$$h = \log_2 \left(\frac{n+1}{2} \right).$$

5. If <B, +, ·, 0, 1> is a Boolean algebra, then for any *two* arbitrary elements $a, b \in B$ prove that :

(*i*) $(a + b)' = a' \cdot b'$

(*ii*) $(a \cdot b)' = a' + b'$

UNIT-III

6. Find the smallest root in magnitude of the equation $x^3 + x^2 + 3x + 4 = 0$, by Chebyshev method of third order. Perform two iterations.

7. Find the values of constants a and b so that the curve $y = ab^x$ fits the given data :

x	y
2	144
3	172.8
4	207.4
5	248.8
6	298.5

UNIT-IV

8. Find the inverse of the matrix :

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

using partition method.

9. Solve the system of equations :

$$83x_1 + 11x_2 - 4x_3 = 95$$

$$7x_1 + 52x_2 + 13x_3 = 104$$

$$3x_1 + 8x_2 + 29x_3 = 71$$

using Gauss-Seidel iteration method.

UNIT-V

10. Given :

$$\frac{dy}{dt} = 1 + y^2,$$

where $y = 0$, when $t = 0$, compute $y(0.2)$ using improved Euler's method.

11. Solve the boundary value problem :

$$y'' = xy$$

$$y(0) + y'(0) = 1 \text{ and } y(1) = 1$$

with step size $h = \frac{1}{3}$.

SECTION-C

12. How many integers are there between 1 and 1000 which are not divisible by 2, 3, 5 or 7?

13. (*a*) Prove that the dual of a complemented lattice is also a complemented lattice.

(*b*) Prove that a tree with n vertices has exactly $(n - 1)$ edeges.

14. Find all the roots of the polynomial equation,

$$x^3 - 3x^2 - 6x + 8 = 0$$

using the Graeffe's root squaring method.

15. Compute the dominant latent root and the corresponding eigenvector of the following matrix :

$$\mathbf{A} = \begin{bmatrix} 1 & 6 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Also compute its other two latent roots.