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M.A./M.Sc. (Sem. IV) Examination, 2020

MATHEMATICS

Paper MATH 4 C(vii)

(Topology—II)

Time Allowed : Three Hours

Maximum Marks : 70

Minimum Pass Marks : 28

*This question paper will be divided into **three** sections as under :*

Section-A ***Max. Marks-10***

*This Section contains **01** compulsory question comprising **10** short answer type questions (maximum **20** words answer) taking two questions from each unit. Each question shall be of **one** mark.*

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Section-B ***Max. Marks-25***

*This Section contains **10** questions, **02** questions from each unit (answer about in **250** words). Students are instructed to attempt **five** questions by selecting **one** question from each unit. Each question shall be of five marks.*

Section-C ***Max. Marks-35***

*This Section contains **five** long answer type questions comprising **one** compulsory question (Question No. 7) of **15** marks **and four** questions of **10 marks** each. Students are instructed to attempt total **three** questions with **one** compulsory question (answer about in **500** words) and any **two** more questions (answer about in **400** words) out of remaining four questions.*

Section A

1. (i) Define sequential compactness.

(ii) Define finite intersection property.

(iii) Define locally connected space.

(iv) What do you mean by component of a space ?

(v) Define product topology.

(vi) Define projection mapping.

(vii) Define Isotone map.

(viii) Define ultranet.

(ix) Show that if a filter F in a topological space X converges to a point $x \in X$, then every filter F′ finer that F also converges to x.

(x) Show that there exists no filter on the empty set ϕ.

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Section B

UNIT-I

2. Prove that a topological space X is compact if and only if every basic open cover of X has a finite subcover.

Or

Prove that a compact subset of a metric space is closed and bounded.

UNIT-II

3. Prove that continuous image of a connected space is connected.

Or

Give an example of a space which is connected but not locally connected.

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UNIT-III

4. Prove that if X be the non-empty product space $X\{X_\lambda : \lambda \in \Lambda\}$, then a non-empty product subset $F = X\{F_\lambda : \lambda \in \Lambda\}$ is closed in X if and only if each F_λ is closed in X_λ.

Or

Prove that the product spaces $X \times Y$ is connected if and only if X and Y are connected.

UNIT-IV

5. If (X, T) be a topological space and let $Y \subset X$ then prove that Y is T-closed if and only if no net in Y converges to a point in X-Y.

Or

Let ψ be an isotone map of directed sets (B, $\geq$) into a directed set (A, $\geq$) such that $\psi[a]$ is cofinal in A. If (*f*, X, A, $\geq$) be a net then prove that $f \circ \psi$ is a subnet of *f*.

UNIT-V

6. Prove that every filter on a set X is contained in an ultrafilter on X.

Or

Let $M = \{F_\lambda : \lambda \in \Lambda\}$ be a non-empty class of filters on a non-empty set X, then prove that M has a supremum if and only if for all finite sub-families $\{IF_i \mid i \leq i \leq n\}$ of elements of M and all $F_i \in IF_i$ $(i \leq i \leq n)$ the intersection $\cap\{F_i \mid i \leq i \leq n\}$ is non-empty.

Section C

7. (*a*) Prove that a metric space X is compact if and only if it is sequentially compact.

(*b*) Prove that every component of a topological space (X, T) is closed. 10+5

8. Prove that a subset E of R is connected if and only if it is an interval. Is R connected ? Justify the answer.

9. Let $\{(X_\lambda : T_\lambda) : \lambda \in \Lambda\}$ be an arbitrary collection of topological spaces and let (X, T) be their product space where $X = X\{X_\lambda : \lambda \in \Lambda\}$, then (X, T) is first countable iff each of the coordinate spaces is first countable and all but a countable number of the coordinate spaces are indiscrete.

10. Prove that a topological space (X, T) is compact if and only if each net in X has a cluster point.

11. (*a*) Let F be a filter on a non-empty set X and let $A \subset X$, then prove that there exists a filter F′ finer than F such that $A \in F'$ if and only if $A \cap F \neq \phi$ for every $F \in$ F.

(*b*) Prove that A net in a set X is ultranet if and only if the filter it generates is an ultrafilter. 5+5