U-1657

M. A. / M. Sc. (Final)
Examination, 2021

MATHEMATICS

Paper - VII

TOPOLOGY

Time allowed : **Three Hours**

Maximum Marks : **100**

This question paper contains ***three*** *sections as under :*

SECTION-A

Max. Marks-10

This section contains **one compulsory** question with 10 parts, *having 2 parts from each unit, short answer in 20 words for each* part. All questions carry **equal** marks.

SECTION-B

Max. Marks-50

This section contains **10** *questions having* 2 questions *from each unit. Answer 5* questions *(250 words each) selecting one* question *from each unit. All questions carry* **equal** marks.

SECTION-C

Max. Marks-40

This section contains ***04*** *descriptive type questions (questions may have sub division) covering all units but not more than one question from each unit. Answer any two questions. (500 words each). All questions carry* ***equal*** *marks.*

SECTION - A

1 (i) Define Topological space.

(ii) What is accumulation point, explain it.

(iii) State briefly open and closed mapping.

(iv) Define Tyconoff space.

(v) What is compactness *in metric space* ?

(vi) State the Heine-Borel theorem.

(vii) Define the separated space.

(viii) What is product space in topology.

(ix) Define the convergence of net.

(x) Explain the intersection of filters.

SECTION - B

UNIT - I

2 Show that union of two topologies for a set 'X' is not necessarily a topology for 'X'.

OR

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3 Prove that every second countable space is first countable.

UNIT - II

4 Prove that a topological space (X, T) is T_1-space iff every singleton subset $\{x\}$ of X is T-closed.

OR

5 Let f be a mapping of (X, T) into (Y, T). then f is continuous iff

$$\overline{f^{-1}(A)} \subset f^{-1}(\bar{A}), \forall A \subset Y.$$

UNIT - III

6 Prove that closed subset of compact sets are compact.

OR

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7 Let A be a compact subset of a metric space (X, d). show that for every $B \subset X$, there exist a point $x_0 \in A$ such that $d(x_0, B) = d(A, B)$.

UNIT - IV

8 Let E be a connected subset of a space X If F is a subset of X such that $E \subset F \subset \overline{E}$ then F is connected.

OR

9 The product space $X = X\{X_\lambda : \lambda \in A\}$ is Hausdorff iff each coordinate space X_μ is Hausdorff.

UNIT - V

10 Let (X, T) be a topological space and let $(f, X, A, \geq)$ be a net in X for each $a \in A$ let $M_a = \{f(x) : x \geq a \text{ in } A\}$ then a point of P of X is a cluster point of f iff $P \in \overline{M}_a \ \forall\, a \in A$.

OR

11 Let C be a non-empty family of subsets of a non-empty set X, then $\exists$ a filter on X containing C iff C has FIP.

SECTION - C

12 A topological space X is countably compact iff every sequence in X has a cluster point in X.

13 Let (X, T) be a typological space and let A be a subset of X. Then A° is the set of all those points of A which are not limit point of A' i.e. $A^0 = \{x \in A : x \notin D(A')\}$.

14 A subset E of R is connected iff it is an interval, in particular R is connected.

15 A topological space (X, T) is Hausdorff iff every net in X can converge to at most one point.