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This question paper contains 8 printed pages]

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M.A./M.Sc. (Sem-II) Examination, 2019

MATHEMATICS

Paper-MATH 2 C(iv)

(Special Functions)

Time Allowed : Three Hours

Max. Marks : 70

Min. Pass Marks : 28

This question paper will be divided into ***three*** *sections as under :*

Section-A ***Max. Marks-10***

will carry ***10*** *marks with* ***1*** *compulsory question comprising* ***10*** *short answer type questions (maximum* ***20*** *words answer) taking* ***two*** *questions from each unit. Each question shall be of* ***one*** *mark.*

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Section-B ***Max. Marks-25***

will carry ***10*** *questions,* ***2*** *questions from each unit (answer about in* ***250*** *words). Students are instructed to attempt* ***five*** *questions by selecting* ***one*** *question from each unit.*

Section-C ***Max. Marks-35***

will carry ***35*** *marks with* ***five*** *long answer type questions comprising one compulsory question (Q. No. 7) of* ***15*** *marks and four questions of* ***10*** *marks each. Students are instructed to attempt total* ***three*** *questions with* ***one*** *compulsory question (answer about in* ***500*** *words) and any* ***two*** *more questions (answer about in* ***400*** *words) out of remaining four questions.*

Section A

1. (*i*) Evaluate :

$$ {}_2F_1(-n, 1; 1; -x). $$

(*ii*) Write the integral representation of ${}_2F_1(a, b; c; x)$.

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(iii) Show that :

$${}_1F_1(a; b; z) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-t} t^{a-1} \, {}_0F_1(-; b; zt)\,dt.$$

(iv) Show that :

$${}_0F_1\left(-;\ \frac{1}{2};\ -\frac{1}{4}z^2\right) = \cos z.$$

(v) Write orthogonality of Bessel function.

(vi) Evaluate :

$$\lim_{x \to 0} \frac{J_n(x)}{x^n}$$

(vii) Write Kummer's confluent Hypergeometric equation. https://www.uokonline.com

(viii) Evaluate :

$${}_1F_1(a;\ a;\ x).$$

(ix) Write the generating relation of Laguerre polynomial $L_n^{(\alpha)}(x)$.

(x) Write the generating relation of Legendre polynomial $P_n(x)$.

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Section B

UNIT-I

2. Prove that :

$${}_2F_1\left(-\frac{n}{2}, \frac{1}{2}(1-n); b+\frac{1}{2}; 1\right) = 2^n \frac{(b)_n}{(2b)_n}$$

$R(b) > 0$ and $n \in N$.

Or

Establish the result :

$${}_2F_1(-n, a+n; c; 1) = (-1)^n \frac{(1+a-c)_n}{(c)_n}.$$

UNIT-II

3. If n is a non-negative integer and if a and b are independent of n, then :

$${}_3F_2\left[\begin{matrix} -n, a+n, \frac{1}{2}+\frac{1}{2}a-b; \\ 1+a-b, \frac{1}{2}+\frac{a}{2}; \end{matrix}\ 1\right] = \frac{(b)_n}{(1+a-b)_n}$$

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Or

If a, b and c, so restricted that each of the function involved exists, then :

$$ {}_3F_2\left[\begin{matrix} a, b, c; \\ 1+a-b, 1+a-c; \end{matrix} 1\right] $$

$$ = \frac{\Gamma\left(1+\frac{1}{2}a\right)\Gamma(1+a-b)\,\Gamma(1+a-c)\,\Gamma\left(1+\frac{1}{2}a-b-c\right)}{\Gamma(1+a)\,\Gamma\left(1+\frac{a}{2}-b\right)\Gamma\left(1+\frac{1}{2}a-c\right)\Gamma(1+a-b-c)} $$

$R(a - 2b - 2c) > -2.$

UNIT-III

4. Prove that :

$$ J_n(x) = (-2)^n x^n \frac{d^n}{d(x^2)^n} J_0(x). $$

Or

Prove that :

$$ x J_n^1(x) = -n J_n(x) + x J_{n-1}(x). $$

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UNIT-IV

5. Prove that :

$$ {}_1F_1(a; c; x) = \frac{\Gamma(c)}{\Gamma(a)\,\Gamma(c-a)} \int_0^1 (1-t)^{c-a-1} t^{a-1} e^{xt}\, dt. $$

Or

Prove that :

$$ (c)_m \frac{d^m}{dx^m}\left[e^{-x} {}_1F_1(a; c; x)\right] $$

$$ = (-1)^m (c - a)_m e^{-x} {}_1F_1(a; c + m; x) $$

UNIT-V

6. Prove that :

$$ e^{2tx-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x). $$

Or

Show that :

$$ \frac{\exp\left\{-\left(\frac{xt}{1-t}\right)\right\}}{1-t} = \sum_{n=0}^{\infty} L_n(x)\, t^n. $$

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Section-C

7. Show that when n is a positive integer, $J_n(x)$ is the coefficient of Z^n in the expansion of $\exp\left\{\frac{1}{2}x\left(z-\frac{1}{z}\right)\right\}$ in ascending and descending power of z.

Also, show that $J_n(x)$ is coefficient of z^{-n} multiplied by $(-1)^n$ is the expansion of above expression.

8. Show that if $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$:

$$\sin mx = m \sin x \; {}_2F_1\left(\frac{1+m}{2}; \frac{1-m}{2}; \frac{3}{2}; \sin^2 x\right)$$

9. If n is a non-negative integer and if b and c are independent of n, then :

$$ {}_3F_2\begin{bmatrix} -n, b, c; & \\ 1-b-n, 1-c-n; & x \end{bmatrix}$$

$$= (1-x)^n \, {}_3F_2\begin{bmatrix} -\frac{1}{2}n, -\frac{1}{2}n+\frac{1}{2}, 1-b-c-n; & \\ 1-b-n, 1-c-n; & -\frac{4x}{(1-x)^2} \end{bmatrix}$$

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10. (a) Show that :

$$\frac{d}{dx}\{{}_1F_1(a; c; x)\} = \frac{a}{c}\,{}_1F_1(a+1; c+1; x)$$

(b) $(a - c + 1)\; {}_1F_1(a; c; x)$

$= a\,{}_1F_1(a + 1; c; x) - (c - 1)\; {}_1F_1(a; c - 1; x)$

11. Show that :

$$(1 - 2xz + z^2)^{-1/2} = \sum_{n=0}^{\infty} z^n p_n(x), |x| \le 1, |z| < 1$$

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