

This question paper contains 4+2 printed pages.

Max. Marks-35

5707

M.A./M.Sc. (Sem. II) Examination, 2018

MATHEMATICS

Paper - Math 2C(ii)

(Real Analysis)

Time Allowed : Three Hours

Max. Marks : 70

Min. Pass Marks : 28

This question paper will be divided into three sections as under :

Section-A **Max. Marks-10**

Will carry 10 marks with 1 compulsory question comprising 10 short answer type questions (maximum 20 words answer) taking two questions from each unit.

Each question shall be of one mark.

Section-B **Max. Marks-25**

Will 10 questions, 2 questions from each unit (answer about in 250 words). Students are instructed to attempt five questions by selecting one question from each unit.

Section-C

Will carry 35 marks with five long answer type questions comprising one compulsory question (question no. 7) of 15 marks and four questions of 10 marks each. Students are instructed to attempt total three questions with one compulsory question (answer about in 500 words) and any two more questions (answer about in 400 words) out of remaining four questions.

Section A

1. (i) Write the necessary and sufficient condition for a function to be Riemann-Stieltjes integrable. <http://www.uokonline.com>
- (ii) Define the upper Riemann-Stieltjes integral.
- (iii) Define pointwise convergence of a sequence of functions.
- (iv) Write the Weierstrass's M-test for uniform convergence.
- (v) Define outer measure of a set $A \subset \mathbb{R}$.

- (vi) Find the Lebesgue measure of the set of integers.
- (vii) Define a Lebesgue measurable function.

(viii) Define equivalent functions on the same set.

(ix) What do you understand by the convergence of a sequence of measurable functions?

(x) Define Lebesgue integral of the function f over the set E .

Section B

UNIT-I

2. Evaluate $\int_0^3 x d[x]$, where $[x]$ is the greatest integer not exceeding x .

Or

Let $f(x) = K$ be a constant function defined on $[a, b]$ and $\alpha(x)$ is a monotonically non-decreasing function on $[a, b]$. Then prove that $f(x)$ is Riemann - Stieltjes integrable.

UNIT-II

3. Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} |x|^n$ is uniformly convergent in $[-1, 1]$.

Or

If $\{f_n\}$ is a sequence of continuous functions on an interval $[a, b]$ and if f_n converges uniformly to f on $[a, b]$, then prove that f is continuous on $[a, b]$.

UNIT-III

4. Prove that the outer measure of a set is translation invariant.

Or

If E is a countable set, then prove that

$$m^*(E) = 0.$$

5. Show that a constant function defined on a measurable set is measurable.

Or

Let f and g be measurable functions defined on a measurable set E . Then prove that $f + g$ is measurable

http://www.uokonline.com

UNIT-V

6. Let f be a bounded measurable function on a measurable set E . then prove that

$$\left| \int_E f(x) dx \right| \leq \int_E |f(x)| dx$$

Or

Let f be a bounded measurable function on a set E and let $f(x) \geq 0$ almost everywhere on E . If $\int_E f(x) dx = 0$, then prove that $f(x) = 0$ almost everywhere on E .

Section C

7. If $f(x)$ is monotonic on $[a, b]$ or $a(x)$ is continuous and monotonic increasing on $[a, b]$, then prove that $f(x)$ is Riemann-Stieltjes integrable with respect to $a(x)$ on $[a, b]$.

8. Test for uniform convergence of the sequence $\langle f_n \rangle$, where $f_n(x) = nx(1 - x)^n$.

9. Prove that every interval is measurable.

10. Prove that there exists a non-measurable set.

11. Let a sequence $\langle f_n \rangle$ of measurable functions defined on a measurable set E converge pointwise to a function f on E . Then prove that the function **f is measurable.**