This question paper contains 4+1 printed pages]

5706

M.A./M.Sc. (Sem. II) Examination, 2019

MATHEMATICS

Paper–Math 2 C(i)

(Advanced Algebra)

Time Allowed : Three Hours

Max. Marks : 70

Min. Pass Marks : 28

This question paper will be divided into ***three*** *sections as under :*

Section-A ***Max. Marks-10***

*Will carry **10** marks with **1** compulsory question comprising **10** short answer type questions (maximum **20** words answer) taking **two** questions from each unit. Each question shall be of **one** mark.*

Section-B ***Max. Marks-25***

*Will carry **10** questions, **2** questions from each unit (answer about in **250** words). Students are instructed to attempt **five** questions by selecting **one** question from each unit.*

Section-C ***Max. Marks-35***

*Will carry **35** marks with five long answer type questions comprising one compulsory question (Question No. **12**) of **15** marks and four questions of **10** marks each. Students are instructed to attempt total **three** questions with **one** compulsory question (answer about in **500** words) and any **two** more questions (answer about in **400** words) out of remaining four questions.*

Section A

1. (i) Define Homomorphism.

   (ii) Write Cauchy Theorem for finite Abelian group.

   (iii) Define subnormal series.

   (iv) Write Jordon Holder Theorem.

   (v) Write Eculidean algorithm for polynomial over a field.

   (vi) Define Eculidean domain.

   (vii) Write unique factorisation theorem.

(*viii*) What is unique factorisation domain ?

(*ix*) What are transcedental extensions ?

(*x*) What is a perfect field ?

Section B

UNIT-I

2. If f is a homomorphism from a group $G \to G'$ with kernel K then $K \Delta G$. Prove.

3. State and prove Cauchy's theorem for finite abelian group.

UNIT-II

4. A normal subgroup M of G is maximal iff the quotient group G/M is simple *i.e.*, M is maximal $\Leftrightarrow \frac{G}{M}$ is simple.

5. There exist at least one composition series for every finite group G. Prove.

UNIT-III

6. The kernel K of a homomorphism f of a ring R to R′ is an ideal of R. Prove.

7. Let $f(x)$ be a polynomial in a polynomial domain $F(x)$ and $a \in F$. Then $f(a)$ is the remainder when $f(x)$ is divided by $x - a$.

UNIT-IV

8. State and prove unique factorisation theorem for Eculidean Rings.

9. Every finite extension of a field is an algebraic extension.

UNIT-V

10. Let K be an extension of a field F. If $a, b \in K$ are algebraic over F of degree m and n respectively and if m, n are relatively prime then prove that $F(a, b)$ is of degree mn over F.

11. Let $a \in K$ be algebraic over F. Then any 2 minimal Monic polynomials for a over F are equal.

Section C

12. (*a*) Every group is isomorphic to some permutation group. Prove.

    (*b*) Any 2 conjugate classes of a group are either identical or disjoint.

13. State and prove Jordan Holder theorem.

14. State and prove second Sylow theorem.

15. (*a*) If $N \Delta G$ such that both N and G/N are solvable, then G is solvable.

    (*b*) Every subgroup of a nilpotent group G is nilpotent.

16. Let K be an extension of a field F then if $a \in K$ is algebraic over F iff $F(a)$ is a finite extension of F. Prove the statement.

http://www.uokonline.com
http://www.uokonline.com