This question paper contains **6** printed pages]

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M.A./M.Sc. (Sem. I) Examination, Dec. 2022

MATHEMATICS

Paper-MATH 1 C4

(Methods of Applied Mathematics-I)

Time Allowed : Three Hours

Maximum Marks : 100

This question paper contains ***three*** *sections as under :*

Section-A ***Max. Marks-10***

This section contains ***one*** *compulsory question with* ***10*** *parts, having* ***2*** *parts from each unit, short answer in* ***20*** *words for each part.* ***All*** *questions carry equal marks.*

Section-B ***Max. Marks-50***

This section contains ***10*** *questions having* ***2*** *questions from* ***each unit****. Answer* ***5*** *questions (****250*** *words each) selecting* ***one*** *question from each unit. All questions carry equal marks.*

Section-C ***Max. Marks-40***

This section contains 4 *descriptive type questions (questions may have* ***subdivision****) covering all units but* ***not more than one*** *question from each unit. Answer any* ***two*** *questions. (****500*** *words each). All questions carry equal marks.*

Section A

1. (i) Define integral equation.

(ii) Define separable kernels.

(iii) Write difference between separable kernel and iterated kernel.

(iv) Define iterated kernel for Fredholm linear integral equation.

(v) If $L\{f(t)\} = F(s-a)$, then find $L\{t^2 f(t)\}$.

(vi) Define complex Fourier transform.

(vii) If $y = y(t)$ and $L\{y\} = \bar{y}$, then find $L\left\{\frac{d^3y}{dt^3}\right\}$ given $y(0) = a = y''(0)$, $y'(0) = b$.

(viii) If $L\{f(t)\} = F(s)$, then find solution of $\int_0^\infty e^{-st} f(t)dt = \frac{1}{s^2}$.

(ix) Write value of $J_{1/2}(x)$.

(x) Express $P_3(x)$ in terms of x.

Section B

UNIT-I

2. Show that the function $g(x) = xe^x$ is a solution of Volterra integral equation :

$g(x) = \sin x + 2\int_0^x \cos(x-t).\ g(t)\ dt.$

Solve the Fredholm integral equation :

$\phi(x) = \lambda\int_0^1 e^{x+t}\phi(t)dt.$

UNIT-II

4. Solve the following integral equation by method of successive approximation :

$g(x) = \left(e^x - \frac{1}{2}e + \frac{1}{2}\right) + \frac{1}{2}\int_0^1 g(t)dt.$

5. Solve by aid of successive approximation :

$g(x) = e^{x^2+2x} + 2\int_0^x e^{(x^2-t^2)}\, g(t)\, dt$

UNIT-III

6. Find Laplace transform of $\sin\sqrt{t}$ and hence deduce the Laplace transform of $\frac{\cos\sqrt{t}}{\sqrt{t}}$.

7. Find Fourier cosine transform of e^{-t^2}.

UNIT-IV

8. Using Laplace transform solve :

$(D^2 + 9)\ y = \cos 2t$, given $y(0) = 1$, $y\left(\frac{\pi}{2}\right) = -1$

9. Using Laplace transform solve the integral equation :

$$g(x) = 1 + \int_0^x \sin(x-t)\, g(t)\, dt.$$

UNIT-V

10. Prove that :

$${}_2F_1\left[\frac{a}{2}, \frac{a}{2}+\frac{1}{2}; \frac{1}{2}; z^2\right] = \frac{1}{2}\left[(1-z)^{-a} + (1-z)^{a}\right]$$

11. State and prove Rodrigues formula for Legendre function.

Section C

12. (a) Solve :

$$g(x) = \lambda \int_0^{2\pi} \sin(x+t)\, g(t)\, dt.$$

(b) Show that the integral equation possesses no solution for $f(x) = x$:

$$g(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi} \sin(x+t)\, g(t)\, dt.$$

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13. Solve :

$$g(x) = \cos x - x - 2 + \int_0^x (t-x)\, g(t)\, dt.$$

14. (a) Find inverse Laplace transform of $\frac{s^2}{s^4 + 4a^4}$.

(b) Using Laplace transform solve :

$ty'' + y' + 4ty = 0$ if $y(0) = 3$, $y'(0) = 0$.

15. State and prove orthogonal property for Bessel's function.