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M.A./M.Sc. (First Semester) Examination, Jan. 2019

MATHEMATICS

Paper Math. 1 C (i)

ADVANCED ALGEBRA–I

(Linear Algebra)

Time Allowed : Three Hours

Maximum Marks : 70

Min. Pass Marks : 28

This question paper will be divided into ***three*** *sections as under :*

Section-A ***Max. Marks-10***

will carry ***10*** *marks with* ***1*** *compulsory question comprising* ***10*** *short answer type questions (maximum* ***20*** *words answer) taking two questions from each unit. Each question shall be of one mark.*

Section-B ***Max. Marks-25***

will carry ***10*** *questions,* ***2*** *questions from each unit (answer about in* ***250*** *words). Students are instructed to attempt* ***five*** *questions by selecting* ***one*** *question from each unit.*

Section-C ***Max. Marks-35***

will carry ***35*** *marks with five long answer type questions comprising one compulsory question (Question No. 7) of* ***15*** *marks and four questions of* ***10*** *marks each. Students are instructed to attempt total* ***three*** *questions with* ***one*** *compulsory question (answer about in* ***500*** *words) and any* ***two*** *more questions (answer about in* ***400*** *words) out of the remaining four questions.*

Section A

1. (i) Define Rank of a linear transformation.

(ii) Define adjoint of a linear transformation.

(iii) Define characteristic equation of a square matrix.

(iv) Show that square matrix A and A^T have the same eigen values.

(v) Define index of Nilpotency.

(vi) Define Diagonalizable Matrix.

(vii) Define symmetric bilinear form.

(viii) Define quadratic form.

(ix) Write Cauchy's Schwarz inequality.

(x) Define orthonormal set.

Section B

UNIT-I

2. If $t : V \to V'$ be a linear transformation, then show that kernel of t is a subspace of V.

Or

Prove that the range and null space of every projection E are T-invariant iff ET = TE.

UNIT-II

3. Find the matrix of a linear transformation t on $V_3(R)$ defined as :

$$t(x, y, z) = (2y + z, x - 4y, 3x)$$

corresponding to the basis :

$$B = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}.$$

Or

Prove that distinct non-zero characteristic vectors of a linear operator t corresponding to distinct characteristic value of t are linearly independent.

UNIT-III

4. Prove that similar matrices have the same minimal polynomial.

Or

Prove that $n \times n$ matrix A over the field F is diagonalizable iff A has n linearly independent eigen vectors in $V^n(F)$.

UNIT-IV

5. If

$$u = (a_1, a_2), v = (b_1, b_2) \in R^2,$$

then prove that the function $h : R^2 \times R^2 \to R$ defined by $h(u, v) = a_1 b_2 - a_2 b_1$ is a bilinear form on $R^2 \times R^2$.

Or

Let

$$\alpha = (x_1, x_2, x_3),\ \beta = (y_1, y_2, y_3) \in v_3(c)$$

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and let

$$f(\alpha, \beta) = 2x_1\bar{y}_1 + (2+3i)x_1\bar{y}_2 + (4-5i)x_1 y_3 + (2-3i)x_2\bar{y}_1 + (6+2i)x_2\bar{y}_3 + (4+5i)x_3\bar{y}_1 + (6-2i)x_3\bar{y}_2$$

be a Hermitian form on $v_3(c)$. Find the matrix of f.

UNIT-V

6. If α and β be any *two* vectors of a real inner product spaces V(R), then prove that :

$$\|\alpha\| = \|\beta\| \text{ iff } (\alpha+\beta) \perp (\alpha-\beta).$$

Also discuss their geometrical interpretation.

Or

If W be any subspace of a finite dimensional inner product space V(F), then prove that :

(a) $V = W \oplus W^1$ where W^1 is an orthogonal complement of W

(b) $(W^1)^1 = W$

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Section C

7. If V(F) and V'(F) are n-dimensional vector spaces having their bases as :

$$B = [b_1, b_2, \ldots\ldots, b_n] \text{ and } B' = [b_1', b_2', \ldots\ldots, b_n']$$

respectively, then prove that there exists a unique linear transformation $t : V \to V'$ such that :

$$t(b_i) = b_i', \quad i = 1, 2 \ldots\ldots n.$$

8. Let V, V', V'' be finite dimensional vector spaces over a field F and let B, B' and B'' be their respective bases then prove that for linear transformation $t : V \to V'$ and $s : V' \to V''$

$$M_{B''}^{B}(sot) = M_{B''}^{B'}(s)\, M_{B'}^{B}(t).$$

9. Let A be a $n \times n$ matrix, then prove that there exists an invertible matrix P such that $P^{-1}AP$ is in the Jordan canonical form.

10. Prove that every quadratic form q over a field F of characteristics not equal to 2 can be diagonalized.

11. Apply Gram-Schmidt process to obtain the orthonormal basis for R^3 whose basis is :

$$\{(1, 0, 1), (1, 2, -2), (2, -1, 1)\}.$$